String-derived renormalization of Yang-Mills theory

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We review the application of bosonic string techniques to the calculation of renormalization constants and effective actions in Yang-Mills theory. We display the multiloop string formulas needed to compute Yang-Mills amplitudes, and we discuss how the renormalizations of proper vertices can be extracted in the field theory limit. We show how string techniques lead to the background field method in field theory, and indicate how the gauge invariance of the multiloop effective action can be inferred form the string formalism. (Proceedings of the 29th International Symposium on the Theory of Elementary Particles, Buckow (Germany), Aug.-Sept. 1995. Preprint DFTT 04/96)

1. Introduction

A string theory contains a parameter having the dimension of a mass squared, called the string tension, and proportional to the inverse of the slope of the Regge trajectory. The field theory or pointlike limit can be obtained by sending the string tension to infinity, or equivalently the Regge slope α' to zero. Performing this limit in the bosonic string one recovers a non abelian gauge theory unified with an extended version of gravity containing also an antisymmetric tensor and a dilaton.

The inverse Regge slope $1/\alpha'$ acts in string theory as an ultraviolet cut off in the integrals over loop momenta, making the string free from ultraviolet divergences.

Because of this it is clear that a string theory can be seen not only as a good candidate for a unified theory of all interactions, but also as a way, through the presence of the Regge slope α', to provide a regularized expression for the amplitudes in gauge theories and gravity.

A very useful feature of string theory for this

purpose is the fact that, at each order of string perturbation theory, one does not get the large number of diagrams characteristic of field theories, which makes it very difficult to perform high order calculations. Using closed strings, one gets only one diagram at each order, while with open strings the number of diagrams remains limited. Furthermore, compact expressions for these diagrams are known explicitly for an arbitrary perturbative order [1], in contrast with the situation in field theory, where no such all-loop formula is known. Finally, string amplitudes are naturally written in a way that takes maximal advantage of gauge invariance: the color decomposition is automatically performed, and so are integrations over loop momenta, so that the helicity formalism is readily implemented.

The combination of these different features of string theory has led several authors [2–7] to use string theory as an efficient tool for computations in Yang-Mills theory. In particular, because of the compactness of the multiloop string expression, it is some times easier to calculate non-abelian gauge theory amplitudes by starting from a string theory, and performing the zero slope

limit, rather than using traditional techniques. In this way the one-loop amplitude involving four external gluons has been computed, reproducing the known field-theoretical result with much less computational cost [8]. Following the same approach, also the one-loop five gluon amplitude has been computed for the first time [9].

The aim of this talk is to summarize some of the results obtained in Refs. [10] and [11]. There it was shown that, provided a simple off-shell continuation is performed, string theory can be used to analyze the structure of ultraviolet divergences in Yang-Mills theory, which arise from string theory in the limit $\alpha' \to 0$. These results complement what is known about the calculation of on-shell scattering amplitudes using strings: in fact, on-shell scattering amplitudes are gauge invariant, while in general renormalization constants are not. For a string-inspired calculation of, say, the β function, or in general of some anomalous dimension, one needs to know precisely in which gauge the calculation is performed, and what regularization prescription and renormalization scheme is being used. Since string theory amplitudes are intrinsecally defined on shell, which gauge and which prescription emerge in the field theory limit will in general depend on how the amplitudes are continued off shell. In fact, while an analysis of the structure of on-shell amplitudes leads to the conclusion that string theory generates a combination of the background field method with the non-linear Gervais-Neveu gauge [12], the only previously known consistent prescription for the off-shell continuation of string amplitudes [13] implies a vanishing wave function renormalization, in contrast with the results of the background field method. This apparent contradiction is solved by adopting a different, and simpler, prescription to go off shell. In the field theory limit the results of the background field method are then recovered, also for gauge-variant quantities such as Z-factors.

As one is starting from an ultraviolet finite theory, it may seem strange that the issue of choosing a regularization prescription should arise at all. However, although $1/\alpha'$ acts as an ultraviolet cutoff, it does not seem practical to use it directly for the renormalization of the field theory that arises when α' is taken to 0. In practice, this would require handling the entire tower of massive string states that can circulate in the loops, whereas one would like to work only with the few states that survive the field theory limit. One must then introduce an auxiliary regularization prescription, so that α' can be eliminated and the analysis performed with a finite number of fields. String theory amplitudes are well-suited to be analytically continued to arbitrary space-time dimension, so it is natural to choose dimensional regularization to handle the divergences that arise when $\alpha' \to 0$. This has the further advantage that the results are directly comparable with most of the perturbative calculations performed in field theory. It should however be kept in mind that this is not the only possibility. In fact one may observe that the "stringy" regularization provided by α' is very close in spirit to Pauli-Villars regularization, as it is constructed by adding to the original theory an infinite number of massive fields, whose masses are then taken to ∞ since they are proportional to the string tension. The coefficients of the various contributions of the massive states are automatically tuned by string theory so that their sum is finite. One might then consider, for example, introducing an effective momentum space cutoff Λ defined to reproduce the finite sum of the massive corrections for finite α'. Divergences would then appear as logarithms of $\Lambda^2 \alpha'$, much as they do in conventional Pauli Villars regularizaztion of electrodynamics, where the electron mass squared plays the role of $1/\alpha'$.

In the following, starting from the one-loop two, three and four-gluon amplitudes in the open bosonic string, and performing the field theory limit, we will show how the renormalization constants Z_A, Z_3 and Z_4 of non-abelian gauge theories can be consistently recovered with a variety of methods. As we shall see, with our prescription string theory leads unambiguously to the background field method.

Before going into the details of the calculation, we want first recall how field theory amplitudes are obtained from string theory, and how we expect those amplitudes to be renormalized.

In field theory one normally computes either connected Green functions, denoted here by

$W_M(p_1 \ldots p_M)$, or one-particle irreducible (1PI) Green functions, $\Gamma_M(p_1 \ldots p_M)$. In both cases, in general, an off-shell continuation is performed, in order to avoid possible infrared divergences.

In string theory, on the other hand, one computes S-matrix elements involving gluon states, which are connected to on-shell connected Green functions truncated with free propagators. Taking the field theory limit, the natural ultraviolet regulator of string theory, $1/\alpha'$, is removed, and, as discussed before, one recovers the unrenormalized Green functions, that are regularized as in field theory by the introduction of the dimensional regularization parameter ϵ. We will see that also in this case an off-shell extrapolation is necessary in order to avoid infrared problems.

Once the field theory limit is taken, it is possible to isolate 1PI contributions, which lead to the 1PI Green functions Γ_M, or to compute the full amplitudes, which lead to the Green functions W_M. From the knowledge on how they renormalize we can then extract the renormalization constants. For example,

$$
\begin{aligned}
\Gamma_2(g) &= Z_A^{-1} \Gamma_2^{(R)}(g) &, \\
\Gamma_3(g) &= Z_3^{-1} \Gamma_3^{(R)}(g) &, \qquad (1.1) \\
\Gamma_4(g) &= Z_4^{-1} \Gamma_4^{(R)}(g) &,
\end{aligned}
$$

while

$$
W_3(g) = Z_3^{-1} Z_A^3 W_3^{(R)}(g) \quad , \qquad (1.2)
$$

where g is the renormalized coupling constant.

The talk is organized as follows. In Section 2 we consider the open bosonic string, and we write the explicit expression of the M gluon amplitude at h loops, including the overall normalization. In Section 3 we give the relevant amplitudes for the tree and one-loop diagrams. In Section 4 we sketch the calculation of the one-loop two gluon amplitude, already presented in [10], and we extract the gluon wave function renormalization constant Z_A. In Section 5 we present an alternative method, that allows one to exactly integrate over the punctures, and we use it to extract the renormalization constants Z_A, Z_3 and Z_4. Finally, in Section 6 we consider an open bosonic string in interaction with an external non

abelian gauge background and, after the integration over the string coordinate, we show how the action for a non abelian gauge field is generated. This leads of course to the same renormalization constants, and perhaps clarifies the connection between string theory and the background field method.

2. The M-gluon h-loop amplitude

In string theory the M-gluon scattering amplitude can be computed perturbatively and is given by

$$
\begin{aligned}
A(p_1, \ldots, p_M) &= \sum_{h=0}^{\infty} A^{(h)}(p_1, \ldots, p_M) \qquad (2.1) \\
&= \sum_{h=0}^{\infty} g_s^{2h-2} \hat{A}^{(h)}(p_1, \ldots, p_M) \,,
\end{aligned}
$$

where g_s is a dimensionless string coupling constant, which is introduced to formally control the perturbative expansion. In Eq. (2.1), $A^{(h)}$ represents the h-loop contribution. For the closed string $A^{(h)}$ is given by only one diagram, while for the open string the number of diagrams is limited in comparison with the large proliferation of diagrams encountered in field theory.

Let us consider the open bosonic string, and let us restrict ourselves only to planar diagrams. For such diagrams the M-gluon h-loop amplitude, including the appropriate Chan-Paton factor, is given by

$$
\begin{aligned}
&A_P^{(h)}(p_1, \ldots, p_M) \\
&= N^h \operatorname{Tr}(\lambda^{a_1} \cdots \lambda^{a_M}) C_h \mathcal{N}_0^M \\
&\quad \times \int [dm]_h^M \left\{ \prod_{i<j} \left[\frac{\exp\left(\mathcal{G}^{(h)}(z_i, z_j)\right)}{\sqrt{V_i'(0)\, V_j'(0)}} \right]^{2\alpha' p_i \cdot p_j} \right. \\
&\quad \times \exp \left[\sum_{i \neq j} \sqrt{2\alpha'} p_j \cdot \varepsilon_i\, \partial_{z_i} \mathcal{G}^{(h)}(z_i, z_j) \qquad (2.2) \right. \\
&\quad \left. \left. + \frac{1}{2} \sum_{i \neq j} \varepsilon_i \cdot \varepsilon_j\, \partial_{z_i} \partial_{z_j} \mathcal{G}^{(h)}(z_i, z_j) \right] \right\}_{\text{m.l.}} \,,
\end{aligned}
$$

where the subscript "m.l." stands for multilinear, meaning that only terms linear in each polariza-

tion should be kept. Eq. (2.2) is written for transverse gluons, satisfying the condition $\varepsilon_i \cdot p_i = 0$, whereas the mass-shell condition $p_i^2 = 0$, though necessary for conformal invariance of the amplitude, has not been enforced yet.

The main ingredient in Eq. (2.2) is the h-loop world-sheet bosonic Green function $\mathcal{G}^{(h)}(z_i, z_j)$, which plays a key role in the field theory limit and is given by:

$$\mathcal{G}^{(h)}(z_i, z_j) = \log E(z_i, z_j) \qquad (2.3)$$
$$- \frac{1}{2} \int_{z_i}^{z_j} \omega^\mu \ (2\pi \mathrm{Im}\tau_{\mu\nu})^{-1} \int_{z_i}^{z_j} \omega^\nu \ ,$$

where $E(z_i, z_j)$ is the prime-form, ω^μ ($\mu = 1, \ldots, h$) the abelian differentials and $\tau_{\mu\nu}$ the period matrix of an open Riemann surface of genus h. All these objects, as well as the measure of integration on moduli space $[dm]_h^M$ for an open Riemann surface of genus h with M operator insertions on the boundary [1], can be explicitly written down in the Schottky parametrization of the Riemann surface, and their expressions for arbitrary h can be found for example in Ref. [14]. Here we give only the explicit expression for the measure of integration in moduli space:

$$[dm]_h^M = \frac{\prod_{i=1}^M dz_i}{dV_{abc}} \qquad (2.4)$$
$$\times \prod_{\mu=1}^h \left[\frac{dk_\mu d\xi_\mu d\eta_\mu}{k_\mu^2 (\xi_\mu - \eta_\mu)^2}(1 - k_\mu)^2 \right]$$
$$\times \ [\det(-i\tau_{\mu\nu})]^{-d/2}$$
$$\times \prod_\alpha {}' \left[\prod_{n=1}^\infty (1 - k_\alpha^n)^{-d} \prod_{n=2}^\infty (1 - k_\alpha^n)^2 \right].$$

where k_μ are the multipliers and ξ_μ and η_μ the fixed points of the generators of the Schottky group. dV_{abc} is the projective invariant volume element

$$dV_{abc} = \frac{d\rho_a \ d\rho_b \ d\rho_c}{(\rho_a - \rho_b) \ (\rho_a - \rho_c) \ (\rho_b - \rho_c)} \quad , \qquad (2.5)$$

where ρ_a, ρ_b, ρ_c are any three of the M Koba-Nielsen variables, or of the $2h$ fixed points of the generators of the Schottky group, which can be fixed at will; finally, the primed product over α denotes a product over classes of elements of the Schottky group [14].

Notice that in the open string the Koba-Nielsen variables must be cyclically ordered according to

$$z_1 \geq z_2 \cdots \geq z_M \quad , \qquad (2.6)$$

and the ordering of Koba-Nielsen variables automatically prescribes the ordering of color indices.

The amplitude in Eq. (2.2) contains two normalization constants which were calculated in Ref. [11], and are given by

$$C_h = \frac{1}{(2\pi)^{dh}} \ g_s^{2h-2} \frac{1}{(2\alpha')^{d/2}} \quad ,$$
$$\mathcal{N}_0 = g_d \sqrt{2\alpha'} \quad , \qquad (2.7)$$

where the string coupling g_s and the d-dimensional gauge coupling g_d are related by

$$g_s = \frac{g_d}{2} (2\alpha')^{1-d/4} \quad . \qquad (2.8)$$

A simple way to explicitly obtain the amplitude $A^{(h)}(p_1, \ldots, p_M)$ is to use the M-point h-loop vertex $V_{M;h}$ of the operator formalism. The explicit expression of $V_{M;h}$ for the planar diagrams of the open bosonic string can be found in Ref. [1]. The vertex $V_{M;h}$ depends on M real Koba-Nielsen variables z_i through M projective transformations $V_i(z)$, which define local coordinate systems vanishing around each z_i, i.e. such that

$$V_i^{-1}(z_i) = 0 \quad . \qquad (2.9)$$

When $V_{M;h}$ is saturated with M physical string states satisfying the mass-shell condition, the corresponding amplitude does not depend on the V_i's. However, as we discussed in Ref. [10], to extract informations about the ultraviolet divergences that arise when the field theory limit is taken, it is necessary to relax the mass-shell condition, so that also the amplitudes $A^{(h)}$ will depend on the choice of projective transformations V_i's, just like the vertex $V_{M;h}$. This is the reason of the appearence of V_i in Eq. (2.2).

3. Tree and one-loop diagrams

For tree-level amplitudes, corresponding to $h = 0$, the various quantities are rather simple. The Green function in Eq. (2.3) is given by

$$\mathcal{G}^{(0)}(z_i, z_j) = \log(z_i - z_j) \quad , \qquad (3.1)$$

while the measure $[dm]_0^M$ reduces to

$$[dm]_0^M = \frac{\prod\limits_{i=1}^{M} dz_i}{dV_{abc}} \quad . \tag{3.2}$$

Inserting Eqs. (3.1) and (3.2) into Eq. (2.2), and writing explicitly all the normalization coefficients, we obtain the color ordered, planar, on-shell M gluon amplitude at tree level

$$
\begin{aligned}
& A_P^{(0)}(p_1,\ldots,p_M) \\
&= 4\,\mathrm{Tr}(\lambda^{a_1}\cdots\lambda^{a_M})\,g_d^{M-2}\,(2\alpha')^{M/2-2} \\
&\times \int_{\Gamma_0} \frac{\prod\limits_{i=1}^{M} dz_i}{dV_{abc}} \left\{ \prod_{i<j}(z_i-z_j)^{2\alpha' p_i\cdot p_j} \right. \\
&\times \exp\left[\sum_{i<j}\left(\sqrt{2\alpha'}\frac{p_j\cdot\varepsilon_i - p_i\cdot\varepsilon_j}{(z_i-z_j)}\right.\right. \\
&\left.\left.\left. + \frac{\varepsilon_i\cdot\varepsilon_j}{(z_i-z_j)^2}\right)\right] \right\}_{\mathrm{m.l.}} ,
\end{aligned}
\tag{3.3}
$$

where Γ_0 is the region defined in Eq. (2.6). Notice that any dependence on the local coordinates $V_i(z)$ drops out in the amplitude when we enforce the mass-shell condition. Notice also that Eq. (3.3) is valid only for $M \geq 3$, since the measure given by Eq. (3.2) is ill-defined for $M \leq 2$.

From the previous equation we can easily derive the three-gluon amplitude

$$
\begin{aligned}
A^{(0)}(p_1,p_2,p_3) &= -4\,g_d\,\mathrm{Tr}(\lambda^a\lambda^b\lambda^c) \\
\times\ &\left(\varepsilon_1\cdot\varepsilon_2\,p_2\cdot\varepsilon_3 + \varepsilon_2\cdot\varepsilon_3\,p_3\cdot\varepsilon_1\right. \\
&\left. + \varepsilon_3\cdot\varepsilon_1\,p_1\cdot\varepsilon_2 + O(\alpha')\right) ,
\end{aligned}
\tag{3.4}
$$

and the four-gluon amplitude

$$
\begin{aligned}
A_4^{(0)}(p_1,p_2,p_3,p_4) &= 4g_d^2\,Tr(\lambda^{a_1}\lambda^{a_2}\lambda^{a_3}\lambda^{a_4}) \\
\times\ &\frac{\Gamma(1-\alpha's)\Gamma(1-\alpha't)}{\Gamma(1+\alpha'u)\,s\,t} \\
\times\ &[(\varepsilon_1\cdot\varepsilon_2)(\varepsilon_3\cdot\varepsilon_4)\,t\,u + (\varepsilon_1\cdot\varepsilon_3)(\varepsilon_2\cdot\varepsilon_4)\,t\,s \\
&+ (\varepsilon_1\cdot\varepsilon_4)(\varepsilon_2\cdot\varepsilon_3)\,s\,u + \ldots] ,
\end{aligned}
\tag{3.5}
$$

where the dots are there to indicate terms of the form $(\varepsilon\cdot\varepsilon)(\varepsilon\cdot p)(\varepsilon\cdot p)$ and terms containing higher orders in α' that we have not explicitly written.

At one loop ($h = 1$) we keep the gluons off the mass shell, and Eq. (2.2) gives, for $M \geq 2$ transverse gluons,

$$
\begin{aligned}
A_P^{(1)}(p_1,\ldots,p_M) &= N\,\mathrm{Tr}(\lambda^{a_1}\cdots\lambda^{a_M}) \\
\times\ &\frac{g_d^M}{(4\pi)^{d/2}}\,(2\alpha')^{(M-d)/2}(-1)^M \\
\times\ &\int_0^\infty d\tau\mathrm{e}^{2\tau}\,\tau^{-d/2}\prod_{n=1}^\infty\left(1-\mathrm{e}^{-2n\tau}\right)^{2-d} \\
\times\ &\int_0^\tau d\nu_M \int_0^{\nu_M} d\nu_{M-1}\ldots\int_0^{\nu_3} d\nu_2 \\
\times\ &\left\{\prod_{i<j}\left[\sqrt{\frac{z_i\,z_j}{V_i'(0)\,V_j'(0)}}\exp\left(G(\nu_{ji})\right)\right]^{2\alpha' p_i\cdot p_j}\right. \\
\times\ &\exp\left[\sum_{i\neq j}\left(\sqrt{2\alpha'}p_j\cdot\varepsilon_i\,\partial_i G(\nu_{ji})\right.\right. \\
&\left.\left.\left. + \frac12\varepsilon_i\cdot\varepsilon_j\,\partial_i\partial_j G(\nu_{ji})\right)\right]\right\}_{\mathrm{m.l.}} ,
\end{aligned}
\tag{3.6}
$$

where $\nu_{ji} \equiv \nu_j - \nu_i$, $\partial_i \equiv \partial/\partial\nu_i$ and τ is related to the period $\tilde\tau$ of the annulus by the relation

$$\tau = -\mathrm{i}\pi\tilde\tau \quad . \tag{3.7}$$

Instead of the Koba-Nielsen variables z_i, we have used the real variables

$$\nu_i = -\frac12\log z_i \quad , \tag{3.8}$$

while the Green function $G(\nu_{ji})$ is given by

$$
\begin{aligned}
G(\nu_{ji}) &= \log\left[-2\pi\mathrm{i}\frac{\theta_1\left(\frac{\mathrm{i}}{\pi}(\nu_j-\nu_i)|\frac{\mathrm{i}}{\pi}\tau\right)}{\theta_1'\left(0|\frac{\mathrm{i}}{\pi}\tau\right)}\right] \\
&\quad - \frac{(\nu_j-\nu_i)^2}{\tau} \quad ,
\end{aligned}
\tag{3.9}
$$

where θ_1 is the first Jacobi θ function.

If we enforce the mass-shell condition $p_i^2 = 0$, any dependence on the local coordinates V_i's drops out. However, in order to avoid infrared divergences, we will continue the gluon momenta off shell, in an appropriate way to be discussed later. Then, following Refs. [10,11], we will regard the freedom of choosing V_i as a gauge freedom. We make the simple choice

$$V_i'(0) = z_i \quad , \tag{3.10}$$

which will lead, in the field theory limit, to the background field Feynman gauge. The conditions (2.9) and (3.10) are easily satisfied by choosing for example

$$V_i(z) = z_i\, z + z_i \quad .\tag{3.11}$$

4. The two-gluon amplitude

The one-loop two-gluon amplitude is given by

$$
\begin{aligned}
A^{(1)}(p_1,p_2) &= N\operatorname{Tr}(\lambda^a\lambda^b)\,\frac{g_d^2}{(4\pi)^{d/2}}(2\alpha')^{2-d/2}\\
&\times\quad \varepsilon_1\cdot\varepsilon_2 p_1\cdot p_2\, R(p_1\cdot p_2)\ ,
\end{aligned}\tag{4.1}
$$

where

$$
\begin{aligned}
R(s) &= \int_0^\infty d\tau\ \mathrm{e}^{2\tau}\,\tau^{-d/2}\prod_{n=1}^\infty\left(1-\mathrm{e}^{-2n\tau}\right)^{2-d}\\
&\times\quad \int_0^\tau d\nu\mathrm{e}^{2\alpha's\,G(\nu)}\,[\partial_\nu G(\nu)]^2\ .
\end{aligned}\tag{4.2}
$$

Notice that if the two gluons are on mass shell, the two-gluon amplitude becomes ill defined, because the kinematical prefactor vanishes, while the integral diverges. In the following we avoid this problem by keeping the two gluons off shell.

To take the field theory limit, we must remember [8] that the modular parameter τ and the coordinate ν are related to proper-time Schwinger parameters for the Feynman diagrams contributing to the two point function. In particular, $t \sim \alpha'\tau$ and $t_1 \sim \alpha'\nu$, where t_1 is the proper time associated with one of the two internal gluon propagators, while t is the total proper time around the loop. In the field theory limit these proper times have to remain finite, and thus the limit $\alpha' \to 0$ must correspond to the limit $\{\tau,\nu\} \to \infty$ in the integrand. The field theory limit is then determined by the asymptotic behavior of the Green function for large τ, namely

$$
\begin{aligned}
G(\nu,\tau) &= -\frac{\nu^2}{\tau} + \log\left(2\sinh(\nu)\right)\\
&- 4\,\mathrm{e}^{-2\tau}\sinh^2(\nu) + 0(\mathrm{e}^{-4\tau})\quad ,
\end{aligned}\tag{4.3}
$$

where ν must also be taken to be large, so that $\hat\nu$ remains finite; in this region, we may use

$$
\begin{aligned}
G(\nu,\tau) &\sim (\hat\nu-\hat\nu^2)\tau - \mathrm{e}^{-2\hat\nu\tau}\\
&- \mathrm{e}^{-2\tau(1-\hat\nu)} + 2\mathrm{e}^{-2\tau}\quad ,
\end{aligned}\tag{4.4}
$$

so that

$$\frac{\partial G}{\partial\nu} \sim 1 - 2\hat\nu + 2\mathrm{e}^{-2\hat\nu\tau} - 2\mathrm{e}^{-2\tau(1-\hat\nu)}\quad .\tag{4.5}$$

We substitute now these results into Eq. (4.1), keeping only terms that remain finite when $k = e^{-2\tau} \to 0$. Divergent terms that correspond to the propagation of the tachyon in the loop must be discarded by hand. The next-to-leading term corresponds to gluon exchange. Since it is also divergent in the field theory limit, the corresponding divergence is regularized by dimensional regularization. Finally, higher order terms $\mathrm{e}^{-2n\tau}$ with $n > 0$ are vanishing in the field theory limit.

By taking the large τ and ν limit we have discarded two singular regions of integration that potentially contribute in the field theory limit, namely $\nu \to 0$ and $\nu \to \tau$ (regions of this type are usually called "pinching" regions, as they correspond to taking the gluon insertions on the worldsheet very close to each other). However, as discussed in Ref. [11], in the case of the two gluon amplitude, these regions correspond to Feynman diagrams with a loop consisting of a single propagator, i. e. a "tadpole". Massless tadpoles are defined to vanish in dimensional regularization, and thus we are justified in discarding these contributions as well.

Replacing the variable ν with $\hat\nu \equiv \nu/\tau$, Eq. (4.2) becomes

$$
\begin{aligned}
R(s) &= \int_0^\infty d\tau\int_0^1 d\hat\nu\ \tau^{1-d/2}\,\mathrm{e}^{2\alpha'\,s\,(\hat\nu-\hat\nu^2)\tau}\\
&\times\quad \left[(1-2\hat\nu)^2(d-2)-8\right]\quad ,
\end{aligned}\tag{4.6}
$$

so that the integral is now elementary, and yields

$$
\begin{aligned}
R(s) &= -\Gamma\left(2-\frac{d}{2}\right)(-2\alpha's)^{d/2-2}\,\frac{6-7d}{1-d}\\
&\times\quad B\left(\frac{d}{2}-1,\frac{d}{2}-1\right)\quad ,
\end{aligned}\tag{4.7}
$$

where B is the Euler beta function.

If we substitute Eq. (4.7) into Eq. (4.1), we see that the α' dependence cancels, as it must. The ultraviolet finite string amplitude, Eq. (4.1), has been replaced by a field theory amplitude which diverges in four space-time dimensions, because of the pole in the Γ function in Eq. (4.7). Defining

as usual a dimensionless coupling constant $g_d = g\,\mu^\epsilon$, with μ an arbitrary mass scale, and having set $d = 4 - 2\epsilon$, we find

$$A^{(1)}(p_1, p_2) = -N \frac{g^2}{(4\pi)^2} \left(\frac{4\pi\,\mu^2}{-p_1 \cdot p_2} \right)^\epsilon \Gamma(\epsilon)$$
$$\times \quad \frac{11 - 7\epsilon}{3 - 2\epsilon}\, B(1 - \epsilon, 1 - \epsilon) A^{(0)}(p_1, p_2) \qquad (4.8)$$

Eq. (4.8)) is exactly equal to the gluon vacuum polarization of the $SU(N)$ gauge field theory that one computes with the background field method, in Feynman gauge, with dimensional regularization, provided we use for the tree-level two-gluon amplitude the expression

$$A^{(0)}(p_1, p_2) \quad = \quad \delta^{ab} [\varepsilon_1 \cdot \varepsilon_2\, p_1 \cdot p_2$$
$$-\varepsilon_1 \cdot p_2\, \varepsilon_2 \cdot p_2] \qquad (4.9)$$

Comparing Eq. (4.8) with the equation for Γ_2 in Eq. (1.1) the minimal subtraction wave function renormalization constant can be extracted

$$Z_A = 1 + N \frac{g^2}{(4\pi)^2} \frac{11}{3} \frac{1}{\epsilon} \quad . \qquad (4.10)$$

In the next section we will recover the previous result for the wave function renormalization constant by means of an alternative method that will also be used for computing the renormalization constants for the three and four-gluon amplitudes. The complete calculation of the amplitude can of course be performed also in these cases, and the result does not change. However, one must then include contributions from pinching regions, which do not vanish, and correspond to one particle reducible diagrams in field theory.

5. An alternative computation of proper vertices

In the previous section we have computed the 1PI two-gluon amplitude and we have extracted the wave function renormalization constant. In this section we present another method, introduced by Metsaev and Tseytlin [2]. This method has the advantage of isolating the 1PI part of the amplitude, and is thus particularly suited to the evaluation of renormalization constants. It is based on the following alternative expression for the bosonic Green function [15]

$$G(\nu_{ji}) \quad = \quad -\sum_{n=1}^{\infty} \frac{1 + q^{2n}}{n(1 - q^{2n})} \qquad (5.1)$$
$$\times \quad \cos 2\pi n \left(\frac{\nu_j - \nu_i}{\tau} \right) \quad + \quad \cdots \quad ,$$

where $q = e^{-\pi^2/\tau}$ and the dots stand for terms independent of ν_i and ν_j, that are not important in our discussion.

The strategy here is different from the one followed when calculating the full amplitude. Since we are only interested in divergent renormalizations, and since pinching singularities will be absent, we can exploit the fact that the power of α' in front of the amplitude is fixed (pinching singularities generate negative powers of α') to discard the exponentials of the Green functions that appear in all amplitudes, and substitute them with a simple IR cutoff. UV divergences will be correctly reproduced since the terms that we are discarding would appear in the integrand raised to the power $d/2 - 2$, and thus would only affect the finite parts.

An important advantage of this approach is that, at least at one loop, it allows to integrate exactly over the punctures before the field theory limit is taken. Since the pinching singularities will be regularized directly in the Green function, we will get for the two gluon amplitude the same expression that we derived in Section 4, while for the three and four gluon amplitudes we will get only the contributions that do not include pinchings and are therefore one-particle irreducible.

Let us start rewriting the one-loop M-gluon planar amplitude as

$$A_P^{(1)}(p_1, \ldots, p_M) = N \operatorname{Tr}(\lambda^{a_1} \cdots \lambda^{a_M})$$
$$\times \quad \frac{g_d^M}{(4\pi)^{d/2}} (2\alpha')^{2 - d/2} (-1)^M$$
$$\times \quad \int_0^\infty d\tau\, e^{2\tau}\, \tau^{-d/2} \prod_{n=1}^{\infty} \left(1 - e^{-2n\tau} \right)^{2-d}$$
$$\times \quad I_M^{(1)}(\tau) \quad , \qquad (5.2)$$

where $I_M^{(1)}(\tau)$ is the integral over the punctures ν_i, and can be read off from Eq. (3.6).

For $M = 2$, after a partial integration with vanishing surface term, we get

$$
\begin{aligned}
I_2^{(1)}(\tau) &= p_1 \cdot p_2 \, \varepsilon_1 \cdot \varepsilon_2 \\
&\times \int_0^\tau d\nu \, (\partial_\nu G(\nu))^2 \left(e^{G(\nu)}\right)^{2\alpha' p_1 \cdot p_2} .
\end{aligned} \tag{5.3}
$$

Using the expression in Eq. (5.1) for the Green function, we can perform exactly the integral over the puncture, and we get

$$
I_2^{(1)}(\tau) = \frac{2\pi^2}{\tau} \, p_1 \cdot p_2 \varepsilon_1 \cdot \varepsilon_2 \sum_{n=1}^\infty \left(\frac{1+q^{2n}}{1-q^{2n}}\right)^2 , \tag{5.4}
$$

This implies that, as far as UV divergences are concerned,

$$
\begin{aligned}
A^{(1)}(p_1, p_2) &= \frac{N}{4} \frac{g_d^2}{(4\pi)^{d/2}} (2\alpha')^{2-d/2} Z(d) \\
&\times A^{(0)}(p_1, p_2) .
\end{aligned} \tag{5.5}
$$

where

$$
\begin{aligned}
Z(d) &\equiv (2\pi)^2 \int_0^\infty d\tau \, e^{2\tau} \, \tau^{-1-d/2} \\
&\times \prod_{n=1}^\infty \left(1 - e^{-2n\tau}\right)^{2-d} \\
&\times \sum_{m=1}^\infty \left(\frac{1+q^{2m}}{1-q^{2m}}\right)^2
\end{aligned} \tag{5.6}
$$

is the integral over the modular parameter that generates the renormalization constants in the field theory limit.

With three gluons we get

$$
\begin{aligned}
I_3^{(1)}(\tau) &= \int_0^\tau d\nu_3 \int_0^{\nu_3} d\nu_2 \Big\{ \varepsilon_1 \cdot \varepsilon_2 \, \partial_1 \partial_2 G(\nu_{21}) \\
&\times [p_1 \cdot \varepsilon_3 \, \partial_3 G(\nu_{31}) \\
&\quad + p_2 \cdot \varepsilon_3 \, \partial_3 G(\nu_{32})] + \dots \Big\} ,
\end{aligned} \tag{5.7}
$$

where terms needed for cyclic symmetry and terms of order α' are not written explicitly, and we discarded the exponentials of the Green functions, as explained above.

The integrals over ν_2 and ν_3 can be done by using the expression in Eq. (5.1) for the Green function. The result is

$$
I_3^{(1)}(\tau) = \frac{(2\pi)^2}{\tau} \, [\varepsilon_1 \cdot \varepsilon_2 p_2 \cdot \varepsilon_3
$$

$$
+ \varepsilon_2 \cdot \varepsilon_3 p_3 \cdot \varepsilon_1 + \varepsilon_1 \cdot \varepsilon_3 p_1 \cdot \varepsilon_2]
$$

$$
\times \sum_{n=1}^\infty \left(\frac{1+q^{2n}}{1-q^{2n}}\right)^2 + O(\alpha') , \tag{5.8}
$$

so that the three gluon amplitude is given by

$$
\begin{aligned}
A^{(1)}(p_1, p_2, p_3) &= \frac{N}{4} \frac{g_d^2}{(4\pi)^{d/2}} (2\alpha')^{2-d/2} Z(d) \\
&\times A^{(0)}(p_1, p_2, p_3) + O(\alpha') .
\end{aligned} \tag{5.9}
$$

Finally, the same calculation can be done for the four-gluon amplitude, where we need to consider only terms whose kinematical prefactor has no powers of the external momenta (and thus is of the form $\varepsilon_i \cdot \varepsilon_j \, \varepsilon_h \cdot \varepsilon_k$). Other terms are suppressed by powers of α'. They are given by

$$
\begin{aligned}
I_4^{(1)}(\tau) &= \int_0^\tau d\nu_4 \int_0^{\nu_4} d\nu_3 \int_0^{\nu_3} d\nu_2 \\
&\Big[\varepsilon_1 \cdot \varepsilon_2 \, \varepsilon_3 \cdot \varepsilon_4 \, \partial_1 \partial_2 G(\nu_{21}) \, \partial_3 \partial_4 G(\nu_{43}) \\
&+ \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_2 \cdot \varepsilon_4 \, \partial_1 \partial_3 G(\nu_{31}) \, \partial_2 \partial_4 G(\nu_{42}) \\
&+ \varepsilon_1 \cdot \varepsilon_4 \, \varepsilon_3 \cdot \varepsilon_2 \, \partial_1 \partial_4 G(\nu_{41}) \, \partial_3 \partial_2 G(\nu_{32}) \Big] .
\end{aligned} \tag{5.10}
$$

Using again Eq. (5.1), we can perform the integrals over the punctures, and we get

$$
\begin{aligned}
I_4^{(1)}(\tau) &= \frac{(2\pi)^2}{\tau} \sum_{n=1}^\infty \left(\frac{1+q^{2n}}{1-q^{2n}}\right)^2 \\
&\times \Big[\varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_2 \cdot \varepsilon_4 - \frac{1}{2} \varepsilon_1 \cdot \varepsilon_2 \, \varepsilon_3 \cdot \varepsilon_4 \\
&\quad - \frac{1}{2} \varepsilon_2 \cdot \varepsilon_3 \, \varepsilon_1 \cdot \varepsilon_4 \Big] .
\end{aligned} \tag{5.11}
$$

The relevant part of the amplitude is then of the form

$$
\begin{aligned}
A^{(1)}(p_1, p_2, p_3, p_4) &= \frac{N}{4} \frac{g_d^2}{(4\pi)^{d/2}} (2\alpha')^{2-d/2} \\
&\times Z(d) A^{(0)}(p_1, p_2, p_3, p_4) + O(\alpha') ,
\end{aligned} \tag{5.12}
$$

where the 1PI part of the four-gluon amplitude at tree level in the background Feynman gauge is given by

$$
\begin{aligned}
A^{(0)}(p_1, p_2, p_3, p_4) &= 4 \, g_d^2 \, \mathrm{Tr}(\lambda^{a_1} \lambda^{a_2} \lambda^{a_3} \lambda^{a_4}) \\
&\times \Big[\epsilon_1 \cdot \epsilon_3 \, \epsilon_2 \cdot \epsilon_4 - \frac{1}{2} \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4 \\
&\quad - \frac{1}{2} \epsilon_2 \cdot \epsilon_3 \, \epsilon_1 \cdot \epsilon_4 \Big] .
\end{aligned} \tag{5.13}
$$

Defining the factor

$$K(d) = \frac{N}{4} \frac{g_d^2}{(4\pi)^{d/2}} (2\alpha')^{2-d/2} Z(d) \quad , \qquad (5.14)$$

we can now perform the limit $\alpha' \to 0$, keeping the ultraviolet cutoff $\epsilon \equiv 2 - d/2$ small but positive, and eliminating by hand the tachyon contribution. The calculation of the integral $Z(d)$ in this limit can be found in Ref. [11]. The result is

$$K(4 - 2\epsilon) \to -\frac{11}{3} N \frac{g^2}{(4\pi)^2} \frac{1}{\epsilon} + O(\epsilon^0) \quad . \quad (5.15)$$

If we finally compare Eqs. (1.1) with Eqs. (5.5), (5.9) and (5.12) we can determine the renormalization constants. They are given by

$$Z_A = Z_3 = Z_4 = 1 + \frac{11}{3} N \frac{g^2}{(4\pi)^2} \frac{1}{\epsilon} \quad , \qquad (5.16)$$

in agreement with the result of the previous section for Z_A, and as dictated by the background field method Ward identities.

6. Open string in an external non abelian background

In this section we will study the interaction of an open bosonic string in an external non-abelian gauge field. In particular we will show that, after the integration over the string coordinate, the leading term of the planar contribution in the small α' expansion reproduces, as expected, the usual gauge invariant Yang-Mills action at each order of string perturbation theory. At one loop, we can also explicitly evaluate its coefficient, reproducing the wave function renormalization constant Z_A, as well as Eq. (5.16). We see that the connection with the background field method is very general, and in fact the gluon amplitudes can be understood as interactions of the string with a particular kind of background, constructed out of plane waves with definite momenta.

Let us consider the planar contribution to the partition function of an open bosonic string interacting with an external non-abelian $SU(N)$ background. It is given by

$$Z_P(A_\mu) = \sum_{h=0}^{\infty} N^h g_s^{2h-2} \int DX Dg \; e^{-S_0(X,g;h)}$$

$$\text{Tr} \left[P_z \exp \left(ig_d \int_h dz \; \partial_z X^\mu(z) A_\mu(X(z)) \right) \right] . \tag{6.17}$$

The path-ordering P_z reminds us that in the open string the z variables are ordered, as in Eq. (2.6), along the world-sheet boundary; it is defined by

$$\text{Tr} \left[P_z \exp \left(ig_d \int_h dz \; \partial_z X^\mu(z) A_\mu(X(z)) \right) \right]$$

$$= \sum_{n=0}^{\infty} (ig_d)^n \int_{\Gamma_{h,n}} \prod_{i=1}^{n} dz_i \tag{6.18}$$

$$\times \quad \partial_{z_1} X^{\mu_1}(z_1) \dots \partial_{z_n} X^{\mu_n}(z_n)$$

$$\times \quad \text{Tr} \left[A_{\mu_1}(X(z_1)) \dots A_{\mu_n}(X(z_n)) \right] \quad .$$

The precise region of integration for the punctures z_i will in general depend on the moduli of the open Riemann surface, and we denoted it by $\Gamma_{h,n}$, for a surface of genus h with n punctures. The gauge coupling constant g_d and the dimensionless string coupling g_s are related by Eq. (2.8). Finally the bosonic string action on a genus h manifold with world sheet metric $g_{\alpha\beta}$ is

$$S_0(X, g; h) = \frac{1}{4\pi\alpha'} \int_h d^2 z \; \sqrt{g} g^{\alpha\beta}$$

$$\times \quad \partial_\alpha X(z) \cdot \partial_\beta X(z) \quad . \tag{6.19}$$

It is convenient to separate the zero mode x^μ in the string coordinate X^μ from the non zero modes ξ^μ through the relation:

$$X^\mu(z) = x^\mu + (2\alpha')^{1/2} \xi^\mu(z) \quad , \tag{6.20}$$

so that ξ^μ is dimensionless, while the zero mode x^μ as well as the string coordinate X^μ have dimensions of length. In terms of the two variables x^μ and ξ^μ the measure of the functional integral in Eq. (6.17) becomes:

$$DX = \frac{d^d x}{(2\alpha')^{d/2}} D\xi \tag{6.21}$$

When we insert in Eq. (6.17) the representation of the path ordered exponential given in Eq. (6.18) and we restrict ourselves to the terms up to the order $O(2\alpha')^2$ we can write the partition function in Eq. (6.17) as follows:

$$Z_P(A_\mu) = \sum_{h=0}^{\infty} \left(\frac{N}{(2\pi)^d} \right)^h g_s^{2h-2} \int \frac{d^d x}{(2\alpha')^{d/2}}$$

$$\times \quad \int d\mathcal{M}_h \Big\{ \mathrm{Tr}(1) - g_d^2 \Big[C_2^{(h)}(A)$$
$$+ \quad \mathrm{i}g_d C_3^{(h)}(A) + (\mathrm{i}g_d)^2 C_4^{(h)}(A) \Big] \Big\} \qquad (6.22)$$

where $C_i^{(h)}(A)$ is the contribution of the terms containing i external gauge fields, and is obtained performing the functional integral over the variable ξ. The measure of integration over the moduli $d\mathcal{M}_h$ is equal to the one given in Eq. (2.4), but does not include neither the differentials of the punctures nor the factor dV_{abc}. For $h > 1$ it includes also the factor dV_{abc} provided that we do not fix any of the punctures. In the case of the tree and one loop diagrams there are not enough moduli to be fixed and therefore another procedure has to be followed, as discussed later.

The calculation of the various terms has been discussed in detail in Ref. [11] and will not be reproduced here. Omitting the vacuum contribution and higher orders in α', we get, for $h > 1$

$$Z_P^{(h)}(A_\mu) = (2\alpha')^{2-d/2} \ N^h g_s^{2h-2} S_h(d)$$
$$\times \quad \int d^d x \left[-\frac{1}{4} F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) \right] , (6.23)$$

where

$$S_h(d) = -g_d^2 \int d\mathcal{M}_h \int dz_1 \int dz_2 \, \theta(z_1 - z_2)$$
$$\partial_{z_1} \mathcal{G}^{(h)}(z_1, z_2) \partial_{z_2} \mathcal{G}^{(h)}(z_1, z_2) , \quad (6.24)$$

and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_d f^{abc} A_\mu^b A_\nu^c . \qquad (6.25)$$

As mentioned before, for $h < 2$ we have to follow another procedure since there are not enough moduli to be fixed and we prefer not to fix any of the punctures in order not to interfere with the definition of path ordering given by Eq. (6.18) and not to significantly complicate the following derivation. However, if we do not fix any of the punctures, we have the problem that at tree and one loop loop level the expressions we write are formally infinite, because we failed to divide by the volume of the projective group. The infinities can be, however, regularized by compactifying the range of integration over the punctures, as discussed in Ref. [17] (see also Ref. [11]). Here we give only the results of the calculations.

At tree level, the projective infinity can be regularized by compactifying the integration region of the variables z_i, mapping them from the real axis to a circle. On a circle, following Ref. [17], we can use the Green function

$$\hat{G}(\phi_1, \phi_2) = \log \left[2\mathrm{i} \sin \left(\frac{\phi_1 - \phi_2}{2} \right) \right] \qquad (6.26)$$
$$= -\sum_{n=1}^{\infty} \frac{\cos n(\phi_1 - \phi_2)}{n} + \dots .$$

The integrals over the punctures ϕ_i are now ordered in the interval $(0, 2\pi)$. The dots in Eq. (6.26) stand for terms independent of the punctures, that are irrelevant.

Using Eq. (6.26), we find that the basic integral appearing in $S_0(d)$ is given by

$$\int_0^{2\pi} d\phi_1 \int_0^{\phi_1} d\phi_2 \ \partial_{\phi_1} \hat{G}(\phi_1, \phi_2) \partial_{\phi_2} \hat{G}(\phi_1, \phi_2)$$
$$= -\frac{2\pi}{2} \int_0^{2\pi} d\phi \sum_{n=1}^{\infty} \sin^2 n\phi$$
$$= -\frac{(2\pi)^2}{4} \sum_{n=1}^{\infty} 1 = \frac{(2\pi)^2}{8} , \qquad (6.27)$$

where we have regularized the sum using ζ-function regularization [2].

The previous result implies that $C_2(A)$ at tree level is equal to [17,11]

$$C_2^{(0)}(A) = -\frac{1}{4} \tilde{F}_{\mu\nu}^a(x) \tilde{F}_{\mu\nu}^a(x) (2\alpha')^2 \frac{(2\pi)^2}{8} . \quad (6.28)$$

where $\tilde{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$.

Similarly we can also compute $C_3^{(0)}(A)$ and $C_4^{(0)}(A)$ in Eq. (6.22) and we can see that the full non-abelian gauge invariant action is correctly reconstructed, in agreement with the results of Ref. [17], where only $C_2^{(0)}(A)$ is explicitly computed while $C_3^{(0)}(A)$ and $C_4^{(0)}(A)$ can be similarly computed without fixing any of the punctures.

The coefficients $C_2^{(1)}(A)$, $C_3^{(1)}(A)$ and $C_4^{(1)}(A)$ can finally also be computed in the case of one loop obtaining the following one-loop contribution to the partition function defined by Eq. (6.17)

$$Z_P^{(h=1)}(A_\mu) = \frac{N}{4} \frac{g_d^2}{(4\pi)^{d/2}} (2\alpha')^{2-d/2} Z(d)$$

$$\times \quad \int d^d x \left[-\frac{1}{4} F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) \right] \;, \qquad (6.29)$$

which is precisely the result of Eq. (5.14), with $Z(d)$ given in Eq. (5.6). We have thus verified that the general formalism developed in the first part of this section applies to the somewhat special cases $h = 0$ and $h = 1$.

7. Concluding remarks

We have shown that it is possible to calculate renormalization constants in Yang-Mills theories using the simplest of string theories, the open bosonic string. This has been done using a variety of methods, and the results concide with the ones obtained using the background field method and dimensional regularization. Since bosonic string amplitudes are well understood at all orders in perturbation theory, this technique may be useful beyond one loop.

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